FORMULA-CONTROLLED TERRITORIAL PUBLIC EXPENDITURE IN THE UNITED KINGDOM*

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Territorial public expenditure allocation within a unitary yet multinational state is a potentially fraught process. Within the United Kingdom, there is a century-old tradition of allocating incremental public expenditure on a formula basis through the Goschen and Barnett formulae. This article mathematically models the Barnett formula and reports numerical simulations. The effects of the formula are demonstrated to be contingent upon, inter alia, the extent of rounding, the price basis of public expenditure planning, and relative population change. Consequently, the intended policy goal of securing some convergence in per capita expenditure relatives may not be achieved. Nevertheless, the resilience of such formula mechanisms attests to their appeal. Greater transparency is likely in the future as the United Kingdom adjusts to a new context of internal devolution and involvement in the European Union project.

I. INTRODUCTION

The literature on fiscal federalism (Musgrave, 1961; Oates, 1972; and King, 1984) has a strongly normative orientation, relating to the optimal tiering and spatial design of government. Much of its development predated the influence of public choice theorists, a factor which probably explains the relatively optimistic view of government which characterizes much of this writing. A revival of interest in this field, where many of the principal results were established some time ago, has paradoxically been occasioned by the reverse process to that normally studied: instead of proposals for decentralizing existing governmental systems, the expected creation out of the pre-Maastricht European Community of some kind of federal European state (Walsh, 1993; Walsh, Reichenbach and Meiklejohn, 1993; Spahn, 1993; and Teute- mann, 1993). In turn, this policy-driven work has stimulated other economists (e.g.,

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Tirole, 1994; and Seabright, 1996) to embark upon a new generation of theoretical inquiry, incorporating recent developments in agency theory.

Political systems allocate money and votes, and it would be surprising if there were not feedbacks between the two processes. Although the United Kingdom is widely viewed as a unitary state and indeed exhibits increasingly centralist fiscal tendencies, territorial minorities believe it constitutes a multinational state. Such asymmetries in perceptions constitute an important dimension of the problem; Scotland and Wales are typically viewed from England as northern and western extensions of England. The United Kingdom was forged by conquest and by accident of royal succession.¹ Unlike Wales, Scotland was never fully integrated into English patterns, having maintained its separate church, law, and administration. Lerais’s (1983) characterization of Scotland as a "nation without statehood" was apt. Despite prolonged debates, which began at the end of the 19th century, Scotland and Wales have not, until now, been the beneficiaries of constitutional reform.²

Nevertheless, there has been decentralized control of much public expenditure, though choice is exercised by "territorial" Secretaries of State³ whose authority derives from the UK Parliament. Shortly after the appointment to the Cabinet in 1885 of a Secretary for Scotland, supported by a Scottish Office, formula mechanisms were developed to regulate changes in public expenditure in Scotland relative to changes in England. Indeed, such formulae for territorial allocation have proved remarkably durable: the Goschen formula (1888 to 1957) and the Barnett formula (1978 to the present) have been in place for 90 out of the last 110 years, surviving many changes of government and huge shifts in public policy. Political controversy about relative public expenditure and taxation quickly degenerates to the level of venomous abuse (Pearce, 1987; and Rosie, 1990): "dependency culture" is set against "colonial exploitation". In contrast, there have been a number of rigorous academic studies of relative public

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¹ Wales was conquered in 1277 and its incorporation into England was fully completed by the Laws in Wales Act 1535. Ireland was conquered in 1649 but not fully incorporated until, following a major rebellion in 1798, the Act of Union (Ireland) 1800 created the United Kingdom of Great Britain and Ireland. Scotland’s history was different: the union of the Crowns occurred in 1603, when James VI of Scotland assumed the English throne as James I. This was followed by the Act of Union 1707, when the two kingdoms came to be represented by a single parliament. The Government of Ireland Act 1920, which provided for separate devolved parliaments in Belfast and Dublin, was implemented only in the north. The south of Ireland seceded in 1922 as the Irish Free State (and changed its name to the Republic of Ireland in 1937). Northern Ireland remained part of what had become, in 1922, the United Kingdom of Great Britain and Northern Ireland. There was devolved government in Northern Ireland from 1921 to 1972.

² Under legislation enacted by the Labour Government elected in May 1997, there will be directly elected devolved bodies in Scotland, Wales and Northern Ireland.

³ There is much political sensitivity concerning how Scotland, Wales and Northern Ireland are described. For that reason, this article follows the Treasury’s bland terminology of "territories". Although England is not usually referred to as a territory, it is convenient in the mathematical modelling to include it within the term.
expenditure and of the processes of territorial allocation (Short, 1978; Borooah and Smyth, 1993; and Heald, 1994). Moreover, renewed constitutional debates in the 1990s about Scotland's position in the United Kingdom have led to the production of better official data about territorial public finances (Scottish Office, 1997b; and Welsh Office, 1997).4

Concerning representation, the cross-party Hansard Society (Chataway, 1991) has drawn attention to the fact that, on a straight population basis, Scotland's parliamentary representation would be 59 instead of 72. On such a basis, Wales would lose five seats (38 down to 33) and England would gain 18 (523 up to 541), with Northern Ireland left unchanged at 17. Population quotas for parliamentary representation were almost identical in England, Ireland, Scotland, and Wales in 1918, but they diverged because of differential population change during the inter-war years, becoming institutionalized when the House of Commons (Redistribution of Seats) Act 1944 established four separate boundary commissions and set the minimum number of Scottish and Welsh seats at 71 and 35, respectively. The White Paper on Scottish devolution (Scottish Office, 1997a) announced that, post-devolution, Scotland's representation in the House of Commons would be determined on the same basis as that for England. This formulation suggests there will be a small number of seats in Scotland beyond population parity, specifically to deal with sparsity and islands. There will be no parallel reduction for Wales, presumably reflecting the Welsh Assembly's lack of legislative powers.

The trends of relative population indicate the long-run tendency for England's relative domination of the United Kingdom to be reinforced. On the basis of official projections, Scotland's share of GB population will be only 8.25 per cent in 2031 (Office for National Statistics, 1998). There has been an almost linear decline in Scotland's population share since the first modern census of 1801. The non-England share of the combined population of (what are now) the United Kingdom of Great Britain and Northern Ireland and the Republic of Ireland fell from 46 per cent in 1821 to 21 per cent in 1991. The underlying socioeconomic forces must have been powerful to have operated so relentlessly at the expense of the peripheral parts of the British Isles for such a long period. Under such circumstances, political conflict as to whether the public policy focus should be upon "people prosperity" or "place prosperity" (Couchene, 1993) is unsurprising.

The specifics of the Goschen and Barnett formulae can be briefly summarized. The Goschen formula allocated changes in certain kinds of public expenditure in the following proportions: England and Wales, 80 per cent; Scotland, 11 per cent; and Ireland, 9 per cent. The original Barnett formula, established in 1978, determined the following proportions: England, 85; Scotland, 10; and Wales, 5. The territorial

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4 It is outside the scope of this article to enter the debate about the direction of net fiscal transfers; informed opinion generally considers that these run from England to the other three territories, but this view is disputed by some commentators in Scotland, notably by those arguing for Scottish independence.
blocks controlled by the territorial Secretaries of State were modified by applying these formula proportions to changes in "comparable" expenditure in England. A supplementary formula gave Northern Ireland 2.75 per cent of changes in comparable expenditure in Great Britain. The Barnett formula was recalibrated in 1992 in recognition of further relative population change as revealed by the 1991 census: the revised proportions were England, 100.00; Scotland, 10.66; and Wales, 6.02; the supplementary Northern Ireland proportion became 2.87 per cent. In preparation for elected devolved bodies in Scotland, Wales, and Northern Ireland, the Barnett formula proportions will, with effect from financial year 1999-2000, be updated annually (Darling, 1997).5

II. MATHEMATICAL FORMULATION OF BARNETT FORMULA

II. A. Preliminaries

The Barnett formula is simple to state, but its implications are complex, particularly because unconnected changes in the public expenditure planning system (such as the introduction of cash planning from 1982-83)6 have fundamentally affected the operation of the formula. A crucial point to note about the operation of the Barnett formula is that there are two components to block expenditure: the inherited expenditure base as at the date of the first implementation of the formula; and the incremental expenditure which has been determined by the operation of the formula. The expenditure relative for "block expenditure" in a territory in a particular year depends upon the relationship between these two components. Thus, initial expenditure advantages which have been "frozen" in the base will depreciate over time as the base comes to constitute a smaller proportion of the block.

Without a counterfactual history of what would have happened without the Barnett formula, it is not possible to make unqualified statements about the effects of the formula. More modestly, however, it is possible to calculate the effect of the operation of the formula on the public expenditure relatives on the basis of particular public expenditure scenarios. The expenditure relative for a particular territory in a particular year will depend upon two factors:

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5 On the first implementation, in July 1998, of this Darling modification, the revised proportions (based on mid-1996 population estimates) were 10.45: 5.95: 100.00, and Northern Ireland 2.91 per cent. These proportions were applied to the three-year forward plans (1999-2000 to 2001-02) established after the Treasury’s Comprehensive Spending Review (Treasury, 1998b). No explanation was provided for the 1992 rebasing on England = 100, which affected only form, not substance.

6 Under cash planning, expenditure plans for later years are expressed in expected outturn prices, and there is a strong presumption against compensation for unexpectedly high inflation. In contrast, the earlier system of volume planning had expressed future years in terms of the prices of the current Survey year; these plans would be revalued each year in line with specific-price inflation.
(a) the inherited expenditure base and the in-built advantage ("excess") or disadvantage ("deficit"), and the speed with which these depreciate; and

(b) the relationship between the initially unchanging proportions\(^7\) of the Barnett formula (10:5:85, corresponding to rounded population percentages in 1976) and the changing population shares.

The period of operation of the Barnett formula can be divided into two sub-periods:

(i) from the adoption of the formula to the last year of volume planning (1981-82); and

(ii) from the adoption of cash planning (1982-83) to the present.

This separation into two sub-periods will serve to demonstrate how sensitive the operation of a seemingly simple formula can be to unrelated changes in the public expenditure planning system.

Moreover, there have been other changes to the conduct of the annual Public Expenditure Survey which have impacted upon the formula, most notably successive redefinitions of the public expenditure control aggregate (Heald, 1995). These are relevant primarily for the way in which they have periodically redefined what was included in block expenditure. Such complications are excluded from the formal modelling, which proceeds on the assumption that each territorial block has a constant composition, and that all expenditure change passes through the formula.

II. B. The Formal Model

In order to develop the argument of Section II.A in a more formal way, the following definitions are made:

\[
c = \begin{cases} 
1 & \text{for England} \\
2 & \text{for Scotland} \\
3 & \text{for Wales} \\
4 & \text{for Northern Ireland}
\end{cases}
\]

(2) \(I_{ct} = \) identifiable expenditure in territory \(c\) during financial year \(t - (t + 1)\) \((1 \leq c \leq 4)\)

(3) \(T_{ct} = \) that part of \(I_{ct}\) which is under the control of the Secretary of State for territory \(c\) \((2 \leq c \leq 4)\)

(4) \(B_{ct} = \) that part of \(T_{ct}\) which is classified as block expenditure \((2 \leq c \leq 4)\)

(5) \(E_{ct} = \) that part of \(T_{ct}\) which is excluded from block expenditure \((2 \leq c \leq 4)\)

(6) \(C_{ct} = \) public expenditure in England on services comparable to those which contribute to \(B_{ct}\) during financial year \(t - (t + 1)\) \((c = 2, 3)\)

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\(^7\) Because the fixed-proportions form of the Barnett formula which operated from 1978 to 1991 nearly polarizes expenditure shares and population shares, the formal exposition is developed in this context. Later, the implications of periodic updating (1992) and annual updating (from 1998) are explored.
It should be noted that $I_{ct}$ is the sum of identifiable expenditure in territory $c$ in year $t$ on a number of programmes, which could themselves be indexed by $j$. However, this is not done in this article because programme numbering and labelling, though vital for empirical work on territorial data over many years, plays no role in the formal model. A crucial obstacle to empirical work on the effects of the Barnett formula is that there are inter-territory differences in which parts of $I_{ct}$ fall within $T_{ct}$, and then in which parts of $T_{ct}$ fall within $B_{ct}$. Moreover, there has been a clear trend for $T_{ct}$ to account over time for a larger proportion of $I_{ct}$, and for $B_{ct}$ to account for a larger proportion of $T_{ct}$. Given the lack of data in the public domain about cross-sectional and time-series differences in block coverage, the working through of formula effects is inevitably obscured (Heald, 1994).\(^8\)

Using these definitions, the formal analysis proceeds on the assumption that there has been a strict application of the Barnett formula; all expenditure changes for the respective blocks are channelled through the formula, and there are no differences between planned and outturn expenditure. Formula bypass could only be modelled if it were to follow systematic rules. Because Northern Ireland fits differently into the Barnett formula,\(^9\) it simplifies the analysis to concentrate on Great Britain (i.e., Scotland, England, and Wales). In order to model the operation of the Barnett formula, let:

\[
\Delta B_{ct} = B_{ct} - \mu_t B_{c(t-1)}
\]

\[
\Delta C_{ct} = C_{ct} - \mu_t C_{c(t-1)}
\]

where the meaning of $\mu_t$ will be clarified shortly. The Barnett formula, which was first implemented in Scotland for financial year 1979-80 (here, dated as $t=1979$) and

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\(^8\) The distinction between identifiable and non-identified expenditure is determined by the Treasury. Specifically, defence expenditure and foreign affairs expenditure are treated as non-identified, on the argument that - wherever incurred - these are undertaken for the benefit of the United Kingdom as a whole. Over time, the proportion of expenditure on other programmes treated as non-identified because of inadequate data has diminished. Although social security is identified to territories, it is excluded from the territorial blocks regulated via the Barnett formula. Although block coverage differs across territories, key components are education, health, and grants to local authorities. In 1996-97 the Treasury (1998a) identified 77% of General Government Expenditure (GGE). In that year, the formula-relevant territorial blocks for Scotland, Wales and Northern Ireland totalled £26 billion, 8% of GGE.

\(^9\) Information about how Northern Ireland fitted into the Barnett formula emerged later than for Scotland and Wales. The linkage of Northern Ireland to changes in Great Britain, rather than in England, probably reflects the particular constitutional circumstances in Northern Ireland. Its devolved Parliament, in operation from 1921-1972, was first suspended and then abolished, because of civil disturbances and terrorist violence. The Barnett formula was conceived in 1978 in relation to Scottish and Welsh devolution and then, after those plans had been abandoned, extended to Northern Ireland, which remained under direct rule. If the coverage of the territorial blocks were identical, it would not matter whether the Northern Ireland block were regulated in relation to comparable expenditure in Great Britain or in England.
in Wales in financial year 1981-82, may be written as the single c-dependent equation:

$$\Delta B_{ct} = \frac{(4 - c)}{17} \Delta C_{ct} \quad \text{for} \ t \geq 1974 + 2c \quad (c = 2, 3).$$

Before proceeding, an explanation of \( \mu_t \) is appropriate. The actual history of the application of the Barnett formula has seen a shift from volume to cash planning. For \( 1979 \leq t \leq 1981 \), annual increments were based on volume planning.\(^{10}\) At \( t = 1982 \) there was a change to cash planning, and cash planning has been in effect since then. Thus the historical \( \mu_t \) is as follows:

$$\mu_t = \begin{cases} 
\text{inflation factor from financial year } (t - 1) - t \\
\text{to financial year } t - (t + 1) & \text{for } 1979 \leq t \leq 1981 \\
\text{conversion factor reflecting change from} \\
\text{volume to cash planning} & \text{for } t = 1982 \\
1 & \text{for } t \geq 1983.
\end{cases}$$

From (7) and (8) we obtain:

$$B_{ct} - \frac{(4 - c)}{17} C_{ct} = \mu_t \left[ B_{c(t-1)} - \frac{(4 - c)}{17} C_{c(t-1)} \right] \quad \text{for } t \geq 1974 + 2c$$

and this may be solved recursively to yield

$$B_{ct} = \frac{(4 - c)}{17} C_{ct} + \left( \prod_{\tau=1974+2c}^{t} \mu_{\tau} \right) \left[ B_{c(1973+2c)} - \frac{(4 - c)}{17} C_{c(1973+2c)} \right]$$

for \( t \geq 1974 + 2c. \)

One way in which to consider the benefits which Scotland and Wales have enjoyed through the implementation of the Barnett formula is by comparing the block expenditure per capita in each of these territories with the corresponding figure for England. Therefore let:

$$P_{ct} = \text{population in territory } c \text{ during financial year } t - (t + 1) \quad (1 \leq c \leq 4)$$

$$Q_{ct} = 100 \left( \frac{B_{ct}/P_{ct}}{C_{ct}/P_{ct}} \right) \quad (2 \leq c \leq 4)$$

From (14) we obtain, for \( c = 2, 3 \):

$$Q_{ct} = 100 \left( \frac{P_{ct}}{P_{ct}} \right) \left\{ \frac{(4 - c)}{17} + \left( \prod_{\tau=1974+2c}^{t} \mu_{\tau} \right) \left[ \frac{B_{c(1973+2c)} - \frac{(4 - c)}{17} C_{c(1973+2c)}}{C_{ct}} \right] \right\}$$

for \( t \geq 1974 + 2c. \)

\(^{10}\) Under volume planning, there was an annual process whereby existing plans were revalued to the new price basis, using specific price indexes relevant to particular programmes (Thain and Wright, 1995).
Since there is a shift to cash planning, so that eventually $\mu_t = 1$, and $C_{ct} \to \infty$ as $t \to \infty$, we can deduce from (12) and (15) that:

\begin{equation}
\lim_{t \to \infty} \frac{B_{ct}}{C_{ct}} = \frac{(4 - c)}{17}
\end{equation}

and

\begin{equation}
Q_{ct} \approx 100 \frac{(4 - c)}{17} \left( \frac{P_{1t}}{P_{ct}} \right) \text{ as } t \to \infty \quad (c = 2, 3).
\end{equation}

Thus, expenditure relatives tend to initial population relatives. However, in the case of Scotland, if its falling relative population were to continue to fall, then (16) implies that eventually:

\begin{equation}
Q_{2t} > 100.
\end{equation}

The block expenditure per capita relative would tend to 100, but for the fact that Scotland's relative population continues to fall.

The term within the square bracket of eq. (15) holds the key to the impact of the Barnett formula on per capita block expenditure relatives. Under cash planning (1982-83 onwards), $\mu_t$ in the leading round bracket = 1. The value of the second round bracket diminishes as both real expenditure growth and inflation increase the value of the denominator. In the case of unchanging relative populations, the block expenditure relatives $Q_{ct}$ would tend to the limiting value of 100. In the case of Scotland, the "population effect" partially offsets the "expenditure effect". In the case of Northern Ireland, increasing population relative to Great Britain means that the population effect reinforces the expenditure effect.

Under the earlier system of volume planning, the revaluation of earlier years' expenditure sustained the value of the second expression in the square bracket, thus "slowing" the process whereby, in the case of unchanging relative populations, block expenditure relatives would tend to 100. The predicted consequences of the Barnett formula were therefore profoundly affected by structural changes in the public expenditure planning system: these were changes entirely unconnected with the mechanics of territorial expenditure determination, the implications for which were almost certainly not fully understood.

II. C. Extensions to the Formal Model

Having established the convergence result and shown how convergence might be reinforced or frustrated by relative population change, there are other features of the model to be explored. In order to keep the notation manageable, two features of the model as developed above are dropped: the $\mu$ variable (for most of the period of
operation of the Barnett formula there has been cash planning); and the device of exploiting the numerical index for c (instead equations are written for each territory). The analysis is developed in terms of the case of Scotland, about which there has been most discussion (Heald, 1992; Bell, et al., 1996).

Consider an idealized system of territorial blocks, in which the coverage of the blocks is identical in all territories and constant through time. Suppose that expenditure on this block in year $t$ is $G_t$, of which $S_t$, $W_t$ and $E_t$ are the block expenditures in Scotland and Wales, and comparable expenditure in England, we assume that $S_o$, $W_o$ and $E_o$ are given, and that all changes in expenditures in subsequent years are determined by the Barnett formula. Therefore:

$\Delta S_t = b_S \Delta G_t \quad (t \geq 1)$

where $b_S$ is a constant. Similar formulae can be written for Wales and England. Given the original proportions of the Barnett formula, it follows that $b_S = \frac{10}{100} = \frac{5}{100}$, and $b_E = \frac{85}{100}$. We assume that relative populations remain unchanged.

By the usual recursive argument:

$S_t = S_o + b_S(G_t - G_o)$.

Let $Q_t^s$ be the index of per capita relative expenditure ($GB = 100$)\textsuperscript{11} for Scotland in year $t$ (that is the expenditure per capita in Scotland expressed as a percentage of the per capita expenditure in Great Britain) so:

$Q_t^s = \left( \frac{S_t/P_s}{G_t/P_G} \right) \times 100$

where $P_S$, $P_G$ are the populations of Scotland and Great Britain. (These variables do not need to be dated because of the assumption of unchanging relative populations.) Combining (20) and (21) gives:

$Q_t^s = \frac{P_G}{P_S} \left[ \frac{S_o + b_S(G_t - G_o)}{G_t} \right] \times 100$

$= \frac{P_G}{P_S} \left[ \frac{S_o + b_S(G_t - G_o)}{G_o + (G_t - G_o)} \right] \times 100.$

\textsuperscript{11} Note that this indexing of per capita relatives on Great Britain reflects the exclusion of Northern Ireland from the formal analysis. Published relatives always index on United Kingdom. Eq. (21) depends on the territorial blocks being identical in programme coverage. In contrast, eq. (14), because it indexes on England, allows for differential coverage.
Moreover, when \( t = 0 \), then:

\[
Q_o^S = \frac{P_G}{P_S} \frac{S_o}{G_o} \times 100.
\]

The amount of Scotland's initial "surplus" eliminated by year \( t \) is:

\[
Q_t^S - Q_o^S = \left\{ \frac{P_G}{P_S} \left[ \frac{S_o + b_S (G_t - G_o)}{G_t} \right] - \frac{P_G}{P_S} \frac{S_o}{G_o} \right\} \times 100
\]

\[
= \frac{P_G}{P_S} \left[ \frac{S_o + b_S (G_t - G_o) - S_o \frac{G_t}{G_o}}{G_t} \right] \times 100
\]

\[
= \frac{P_G}{P_S} \left[ b_S \frac{(G_t - G_o)}{G_t} + S_o \frac{(G_o - G_t)}{G_t} \right] \times 100.
\]

Therefore:

\[
Q_t^S - Q_o^S = \frac{P_G}{P_S} \left( b_S - \frac{S_o}{G_o} \right) \left[ \frac{(G_t - G_o)}{G_t} \right] \times 100.
\]

Assuming \( \frac{S_o}{G_o} > b_S \) (which reflects present reality) and assuming \( G_t > G_o \), we can see that \( Q_t^S < Q_o^S \) for \( t \geq 1 \), which means that Scotland's relative per capita expenditure will reduce. Eq. (28) gives a measure of the reduction at any time \( t \). Analogous equations can be derived for England and Wales, in the former of which cases relative per capita expenditure will increase.

In Section III, Figs. 2 and 5 will show, for the "exact" and "rounded" cases, the elimination paths for excesses and deficits. To avoid breaking the flow, a brief explanation is provided here, with the full analysis being contained in the Appendix. When there is full convergence, the elimination paths of the excess/deficit are identical for each territory. When there is not full convergence (because the original formula proportions did not exactly match population shares), this result breaks down. What is identical are the paths from each starting point to the particular relative to which partial convergence is tending. Another way of expressing this is that, when 50 per cent of that part of Scotland's excess which will be eliminated has been eliminated, 50 per cent of the counterpart excess for Wales has also been eliminated, as has 50 per cent of the counterpart English deficit.

II. D. Summary of Theoretical Predictions of the Impact of the Barnett Formula

The formal mathematical modelling allows the following propositions to be stated:
Proposition 1: For all three non-English territories, the proportions of the Barnett formula were substantially less favourable than the relatives incorporated in the inherited expenditure base. Therefore, the continued strict application of the formula would set in motion a long-term tendency towards convergence of expenditure relatives to the UK average, provided that there was growth in the expenditure aggregates to which the formula was applied.

Corollary 1: In contrast, if these expenditure aggregates became smaller, the imposition of expenditure reductions in proportion to population rather than to expenditure would magnify the dispersion of relatives (Heald, 1980). This was indeed the context in which the existence of the Barnett formula became public knowledge in late 1979; the incoming Conservative Government was seeking reductions in the volume of public expenditure, the basis on which public expenditure planning was then conducted.

Proposition 2: The population rounding inherent in 10:5:85 (Barnett) and 2.75:100 has a significant effect. Taking 1976 populations, Scotland’s share of GB population was 9.57 per cent (10 was a rounding up); Wales’ was 5.12 per cent (5 was a rounding down); and England’s was 85.31 per cent (85 was a rounding down). Obviously, the smaller the population proportion, the greater the potential rounding error. The ratio in 1976 of Northern Ireland’s population to GB population was 2.79 per cent (2.75 per cent was a rounding down).

Proposition 3: The "frozen-base advantage" depreciates as the proportion of "inherited" block expenditure diminishes as a proportion of total block expenditure. Consequently, the switch to cash planning for 1982-83 onwards was profoundly important because the inherited base ceased to be revalued to the current year’s price level and would thus fall more rapidly as a proportion of the block total.

Corollary 2: An unexpected result of the formal analysis is that, in the case when the formula proportions exactly equal population proportions, and relative proportions never change, the rate of elimination of initial excesses and deficits is identical in all four territories. This and the result for rounded populations are set out in the Appendix.

Proposition 4: Provided that the Barnett formula continues in operation, relative population decline will, to a greater or lesser extent, offset the "incremental expenditure" convergence effect. In the long run, it was unlikely that the Barnett formula would hold, at least in its fixed-proportions form, if there were marked changes in relative populations. In the short run, the view adopted about this "benefit" to Scotland will depend, inter alia, upon whether the focus is upon total Scottish welfare or per capita Scottish welfare. Whether higher relatives imply higher levels of service would depend upon the relationship between fixed and variable costs of provision, and upon utilization rates. It is important to stress that these are statements about long-run tendencies, given the consistent application of an unchanged formula and the absence of bypass. If bypass is ad hoc rather than systematic (in the sense of following some
rule), important caveats have to be entered. Even with strict application, these results would unfold over a number of years.

Proposition 5: If the formula had exactly reflected population shares and these had remained constant through time and there was no bypass, the percentage increases in English equivalent expenditure would be higher than the formula-driven percentage increase in the Welsh block. In turn, the Welsh percentage would be higher than the formula-driven percentage increase in the Scottish block. This is an arithmetical result of the formula consequences for Scotland and Wales being generated by the lower English expenditure base, and of Scotland's expenditure differential over Wales. Accordingly, it is incorrect to examine percentage changes in the Scottish and Welsh blocks and then to suggest that higher Welsh percentage increases indicate a breach of the Barnett formula. Such differentials are a natural outcome of the distinctive treatments of the expenditure base and expenditure increment. To complicate matters, the relationships between formula-driven percentage increases have in practice been significantly modified by the extent of rounding of base-year population shares inherent in the original choice in 1978 of 10: 5: 85. In contrast, the recalibration in 1992 to 10.66: 100.00: 6.02 involved much less rounding, as two places of decimals were used instead of only integers.

Corollary 3: Annual updating (as from 1998), and to a lesser extent periodic updating (as in 1992), restore the result of full long-term convergence of per capita relatives. Nevertheless, even continuously updated formula proportions only affect the annual increments, whilst per capita relatives are calculated by dividing current-year expenditure (most of which depends on expenditure inherited from the past) by current-year population. Accordingly, the paths of expenditure relatives will depend on the paths of nominal expenditure (since 1982-83, previously real expenditure) and relative population.

III. SIMULATIONS OF THE BARNETT FORMULA

Simulations of the formal model of the Barnett formula are a convenient way of presenting the results. In order to emphasize the direction of the changes, the magnitude of expenditure and population changes is necessarily exaggerated. It is not suggested that the Barnett formula would in practice be left in place unchanged whilst these effects worked through; rather, the intention is to illustrate visually the internal dynamics of the formula. It should be stressed that Figs. 1 to 7 graph simulations of the strict operation of the Barnett formula; thus, all expenditure change is passed through the formula and there is no bypass. Simulations are presented for the original

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12 One might imagine circumstances in which the different coverage of the Scottish and Welsh blocks might disrupt this result, but the expenditure differentials seem sufficiently large to rule out such an outcome.
Barnett formula proportions (10: 5: 85 and 2.75 per cent), thereby including Northern Ireland.

Fig. 1 plots changes in per capita relatives on the counterfactual assumption that the original Barnett formula proportions of 10: 5: 85 and 2.75 per cent exactly represented relative populations in the base year. In order to avoid differences between formula proportions and population proportions affecting this particular simulation, GB population at mid-year 1979 has been reallocated to the three territories of Great Britain on the Barnett proportions, and the UK population then redetermined using the Northern Ireland formula proportion. Block expenditure in 1979-80 is then recalculated for adjusted populations so that per capita block expenditure relatives are unchanged. The horizontal scale shows 10,000 rounds each of an incremental £100 million for Great Britain; Scottish increases in the block are £10 million for each increment and £100,000 million cumulatively. The equivalent applies to Wales and Northern Ireland population shares, which are maintained constant throughout the simulation at the exact formula proportions. This simulation of what might be described as the pure form of the fixed-proportions Barnett formula (i.e., formula proportions are the base-year population shares) shows the predicted convergence of the per capita relatives to 100.

**FIGURE 1**
Block Relatives
10:5:85 as exact population shares

Fig. 2 graphs the elimination of excesses or deficits in per capita relatives through the operation of the original formula proportions, again on the counterfactual assumption that these were the actual population proportions in the base year. An unexpected result is that the rate at which the initial excess/deficit is eliminated is the same for all the territories. The early stages of the convergence are shown to be quite rapid. By
round 350 (i.e., a GB increment of £35,000 million in relation to an adjusted GB base of £37,819 million), 51 per cent of the initial excess/deficit has been eliminated. A point to note is that, though the horizontal axis again measures successive increments
of £100 million in total "block-equivalent" GB expenditure, it is re-interpreted as cumulative increments as a percentage of the "GB block".

Fig. 3 uses the original formula proportions and again makes the counterfactual assumption that these represent the exact population proportions in the base year. Relatives are plotted between the percentage increases in the Scottish block relative to each of the other territorial blocks. When governments make their public expenditure announcements, comparisons are naturally made between the percentage increases registered on various programmes. On the assumption that the Barnett proportions did exactly reflect 1979 population shares and that relative populations remain constant, the percentage increase in the Welsh block would always exceed that in the Scottish block. This reflects the fact that increments were allocated on a population basis, whereas the inherited expenditure base has been more favourable to Scotland. In contrast, Scotland’s percentage increase starts considerably higher than that for Northern Ireland. These differences lessen as formula-induced convergence occurs.

**FIGURE 4**

Block Relatives
10:5:85 as rounded population shares

Fig. 4 graphs the per capita relatives, but this time starting the simulation from the actual 1979 population shares whilst using the Barnett proportions to allocate incremental expenditure. The process of convergence is now more complex. Scotland converges not on 100 but on 104.53, reflecting the extent to which the Barnett proportion of 10 was more favourable than its 1979 population share of 9.57 per cent. Wales converges on 97.69, reflecting the adverse rounding of its Barnett proportion. In consequence, Wales, though starting from a per capita index above 100, actually falls below 100, as does Northern Ireland (which converges on 98.60). England converges...
on 99.68, reflecting the smaller impact of adverse rounding. The rounding inherent in 10:5:85 has exerted a pronounced effect.

Fig. 5 graphs the elimination of excesses or deficits in per capita relatives through the operation of the original formula proportions, using the actual population proportions in the base year. The recognition that the Barnett formula used rounded, not exact, population shares results in the rate at which the initial excess/deficit is eliminated differing for each territory. The initial advantage to Wales is eliminated, then substantially overeliminated. In contrast, Scotland’s initial advantage is never fully eliminated.

Fig. 6 replays the simulation of relative change in block expenditure, on the basis that the Barnett proportions were rounded approximations of relative populations. The lines have the same general shape as their counterparts in Fig. 3 but the vertical intercepts are further from 100, reflecting the way in which the favourable rounding inherent in the original formula proportions partially offsets the predicted lower percentage increases for Scotland.

Per capita indexes are, of course, affected by changes in population as well as in expenditure. The simulations reported so far have assumed unchanged relative populations at the 1979 level. In reality, however, Scotland’s population has continued its long-term absolute decline, thus leading to a more marked relative decline because the absolute populations of the three other territories have been increasing. Relative populations have changed since 1979, with Wales’s and Northern Ireland’s rising and Scotland’s falling, thus mitigating the convergence effect of the Barnett formula on the latter’s per capita relative.
Simulations in the context of changing relative populations can only be illustrative, as they must rest on hypothetical scenarios of relative population change. In the simulation underlying Fig. 7, Scotland's adjusted (to match the original Barnett proportions) GB population share started as 10.00 per cent, but at each successive 50th
simulation it is arbitrarily assumed to fall by 0.01 percentage points (i.e., 9.99 per cent, 9.98 per cent, etc.); this drop in Scottish population is transferred to England, leaving Wales and Northern Ireland unchanged. These figures are solely intended to be illustrative of the processes involved. A sufficiently large drop in Scotland’s relative population could stop - and indeed reverse – the process of convergence of per capita relatives.

IV. A HISTORICAL PARALLEL: GOSCHEN AND BARNETT

There are remarkable echoes between this account of the Barnett formula and Mitchell’s (1991) account of the earlier Goschen formula. Until Mitchell’s research, the practical application of the Goschen formula had been astonishingly badly documented. Despite the dramatic changes in the public expenditure context, the continuing relationship between Scotland and England within the United Kingdom means that there are interesting parallels and contrasts between the two formulae.

First, the orthodox account of the origins of the Goschen formula, usually derived from Boyle (1966), attributes its proportions to relative population shares. Using census population data reported by the Royal Commission on Local Taxation (Balfour of Burleigh, 1902), Mitchell demonstrated that this cannot have been the case. Instead, the Goschen formula of Scotland (11 per cent), Ireland (9 per cent) and England and Wales (80 per cent) represented the assignment of probate duties in the (rounded) percentages of their overall contributions to the Exchequer. The Goschen formula remained in use until 1957, most notably in the context of educational grants.

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13 Dealing systematically with relative population change renders the mathematics more complicated than in this article, and reduces the expositional value of graphed simulations. The problem is that the effects of relative population change can only be handled by a full specification of the time paths of expenditure and of relative populations.

14 This section could not have been written without pre-publication access to a draft chapter from a book on the Scottish Office, being written by Professor James Mitchell of the University of Sheffield (Mitchell, 1991).

15 The account in Heald (1980) is incorrect; see, instead, the explanation in Heald (1992, pp. 54-57).

16 In these discussions, references were made to three territories, Wales being assumed to be part of England. If probate duty had been assigned on the basis of its territory of origin, the proportions would have been: Scotland, 10 per cent; Ireland, 5 per cent; and England and Wales, 85 per cent (Goschen, 1888a). Population shares in 1881 were: Scotland, 10.7 per cent; Ireland, 14.8 per cent; and England and Wales, 74.5 per cent (Mitchell, 1991). Goschen later refused a request by Sir George Campbell MP to publish “in a return” the relevant figures on which the Goschen formula was based (Goschen, 1888b).

17 "After 1918, the Goschen formula had annually and automatically given Scotland eleven-eightheths of the amount by which the annual grant for England and Wales exceeded the grant that had been given in 1913-14" (McPherson and Raab, 1988, p. 178). The abolition of the Goschen formula seems to have caught the then Secretary of State for Scotland (John Maclay) by surprise. Having on 12 February 1957 been able to see “no possible reason why the Goschen formula should be affected” by proposed changes to local government grants (Maclay, 1957a), on 17 December 1957 he announced its demise because of
Initially, it was less favourable to Scotland than population shares, but it then became very similar to population shares, and latterly was more favourable as Scotland’s relative population continued to decline.

Second, Goschen had decided to introduce Local Taxation Accounts for Scotland, Ireland, England, and Wales which would be fed by certain assigned revenues, as part of an attempt to separate "Imperial" from "Local" taxation (Watson Grice, 1910). In a very different context, the Barnett formula was also conceived as an expenditure-regulating mechanism, designed to limit conflict in the financial relationships between the (then proposed) Scottish and Welsh Assemblies and the UK Treasury.

Third, one reason why it has proved so formidably difficult to document the use of the Goschen formula is that it did not have universal application. Indeed, whether to use it in particular instances proved a highly political question, with opportunism being the dominant motive:

... the Treasury was consistent only in ensuring that expenditure was limited ... The Scottish MPs who were most vociferous on the matter were as inconsistent as the Treasury regarding the means of distribution involved but, mirroring the Treasury, consistent in supporting whichever method gave most to Scotland in any given situation (Mitchell, 1991, p. 16).

Subject to the caveats about bypassing Barnett, the new formula has been systematically applied across a much wider range of expenditure.

Fourth, Mitchell’s account of the application of the Goschen formula over its 70-year life highlighted the four principal bases upon which expenditure might be allocated to each territory:

(i) by revenue raised from the particular tax under consideration (i.e., by a strict application of the derivation principle);

(ii) by overall contributions to tax revenue;

(iii) by relative need for particular types of expenditure; or

(iv) by a durable, pre-set formula.

The Goschen formula was originally based on a form of derivation, though a strong element of pro-Ireland redistribution was built in by the decision to assign probate duties not according to its own proportions but according to the overall proportions of Exchequer revenue contributed by each territory (i.e., basis (ii) rather than (i)). In practice, both Goschen (in the way it came to be used) and Barnett (from the beginning) were examples of (iv). A strict application of the Barnett formula would always reject (iii), though some of the identified cases of bypass18 involve an ad

the disappearance of an "identifiable English grant in aid of education to which the Goschen fraction can be applied" (MacKay, 1957b).

18 When, as on health, the territorial expenditure relative is substantially above UK = 100, this is likely to reflect in part a higher per capita employment of nurses. If the Treasury were to underwrite the full cost to each health department of a UK nurses’ pay settlement, the territories would receive more than if the total UK cost of the award were to be distributed through the Barnett formula.
hoc recourse to a form of (iii). Whereas the application of Goschen was piecemeal, with decisions being taken on a case-by-case basis as to whether it would be applied, Barnett has been used extensively for a defined set of services for which – because social security is excluded – it can reasonably be argued that relative need is unlikely to be volatile from year to year.¹⁹ A long-standing tension surrounding the application of the Goschen formula to Scottish education was the fact that, in terms of participation rates in secondary education, Scottish provision was more extensive than that in England. This issue reappeared in the 1990s owing to the "repatriation" of the Scottish universities, hitherto financed on a GB basis via the Universities Funding Council (UFC). As from April 1st, 1993, the Scottish Higher Education Funding Council took over the Scottish Office-financed Central Institutions (non-university providers of higher education) and the UFC’s Scottish responsibilities. A negotiated amount, based upon UFC’s 1993-94 planned expenditure on Scottish universities, was transferred into the Scottish block (i.e., not 10.66 per cent of English expenditure). The Scottish block then received incremental expenditure deriving from the formula consequences of English incremental expenditure, leading to a measure of financial strain on the block, as a result of much higher Scottish participation rates than in England.

Fifth, both the pre-1992 Barnett formula and particular applications of the Goschen formula involved two elements: (a) "frozen" expenditure bases as at a particular year; and (b) the application of a pre-set formula for marginal changes. Mitchell (1991) recounted the successful struggle of the Scotch Education Department to have 1913-14 chosen as the base year rather than 1911-12 when the Goschen formula was to be adopted as the basis for the calculation of grants under the Education (Scotland) Act 1918; this arrangement gave Scotland an advantage, albeit a depreciating one, which survived until the abandonment of Goschen in 1957 (McPherson and Raab, 1988).

Sixth, in the case of both Goschen and Barnett, the question has been raised as to whether such formula-linking to English expenditure deprives Scotland of policy initiative. Ronald Munro-Ferguson, then Liberal MP for Leith Burghs, but later to become, as Viscount Novar, Conservative Secretary of State for Scotland (1922-24), complained in 1912:

Scotland is simply tied to the tail of England, and she is denied all initiative. She becomes the dumping ground for some equivalent grant when expenditure is undertaken in England for some purpose that in Scotland is practically uncalled for (quoted by Mitchell, 1991, p. 21).

Leaving aside the colourful imagery, such asymmetry is arithmetically inevitable given the relative population sizes of Scotland and England. Under Goschen, an £80 increase in English and Welsh expenditure would facilitate additional Scottish expenditure of £11 and Irish expenditure of £9. Under a "reverse" Goschen, an additional £11

¹⁹ In a UK context, social security expenditure excludes health. A significant development has been the transfer into the territorial blocks of economic development expenditure, including industrial assistance, for which the assumption of reasonably stable relative needs might not hold.
of Scottish expenditure would facilitate additional English and Welsh expenditure of £80, and additional Irish expenditure of £9. The multipliers are, respectively, 25 per cent and 400 per cent. Under the original Barnett formula, an additional £85 of English expenditure would generate additional Scottish expenditure of £10, additional Welsh expenditure of £5, and additional Northern Ireland expenditure of £2.75. With a Barnett formula driven by Scotland, additional Scottish expenditure of £10 generates additional English expenditure of £85, Welsh expenditure of £5, and Northern Ireland expenditure of £2.75. The multipliers are, respectively, 20.88 per cent and 927.50 per cent. No UK Treasury could ever be unaware of this asymmetry: equivalent provision for Scotland, Wales, and Northern Ireland involves a modest increment over the English cost, whereas equivalent provision for England, Wales, and Northern Ireland would involve a huge budgetary cost. Undoubtedly, two factors have enormously softened this adverse consequence for Scotland of formula-linking to England: the persistent expenditure differential in favour of Scotland which the Goschen and Barnett formulae have protected for roughly 90 of the last 110 years; and the extensive expenditure-switching discretion over their blocks secured by the territorial departments in the 1980s, for which no counterpart has ever existed in England – either for the whole territory or for individual regions. Whereas the combination of formula allocation and expenditure-switching accords recognition to Scotland, Wales, and Northern Ireland as units for political decision-making, no such recognition is accorded to England (or to English regions).

V. CONCLUSION

This analysis of the UK experience of territorial allocation within a unitary yet multinational state has shown that there has been a strong revealed preference in favour of formula mechanisms. Rather than attempt a summary of detailed findings, the article will conclude with four observations about formula longevity and the implications of devolution.

First, there has been a remarkable lack of transparency. Although the Goschen and Barnett formulae are themselves simple, tracking their effects can be extremely complex. Even if the full logical implications of the formulae had been understood, the relevant data have never been in the public domain. Moreover, it is highly doubtful whether relevant data have been kept within government in a form which would permit systematic tracking.

Second, this investigation of the properties of the Barnett formula has shown that mechanism inertia does not necessarily lead to outcome inertia. The path of per capita expenditure relatives depends crucially upon expenditure growth and relative population change. All predictions about effects must therefore be contingent on such variables.
Third, the longevity of these formulae may be partly attributable to their usefulness as a device for reducing transaction costs, most notably because the element of automaticity limits bureaucratic conflict on sensitive issues of territorial politics. This may have been appealing because, even taken together, the three territorial blocks regulated by the Barnett formula account for only a relatively small percentage of GGE. Furthermore, the territorial departments have strong incentives to increase the value-for-money of block expenditure because they can redeploy such savings within the block.

Fourth, the Labour Government's plans for elected devolved bodies in Scotland, Wales, and Northern Ireland have spectacularly raised the political profile of the Barnett formula itself and of territorial expenditure relatives more generally (Treasury Committee, 1997; Heald et al., 1998). Although these plans are predicated upon the continued use of the Barnett formula, several commentators have doubted the viability of this approach. The adoption of the Barnett formula in 1978 occurred when the results of the Treasury's (1979) expenditure needs assessment study were already known within government. The Labour Government has refused to commit itself to a new needs assessment study, though one may be conducted after the devolved bodies become operational. Undoubtedly, there will be a marked change of context when the Barnett formula stops being a mechanism internal to one government and becomes a mechanism between governments (which might well have different political complexions).

APPENDIX

This Appendix justifies the results stated towards the end of Section II.C. Two cases are distinguished.

Case 1 - 10: 5: 85 as exact population shares

In this case $P_t : P_W : P_E = 10 : 5 : 85$. Therefore, Eqs. (23) and (25) become:

\[ Q^s_t = \frac{100}{10} \left[ \frac{S_o + \frac{10}{100} (G_t - G_o)}{G_o + (G_t - G_o)} \right] \times 100 \]  

(29)

\[ Q^s_t - Q^s_o = \frac{100}{10} \left( \frac{10}{100} - \frac{S_o}{G_o} \right) \left( \frac{G_t - G_o}{G_t} \right) \times 100. \]  

(30)

Note that we can express the index of relative per capita expenditure as a function of the increment of expenditure $Z$, so that

\[ Q^s = \frac{100}{10} \left( \frac{S_o + \frac{10}{100} Z}{G_o + Z} \right) \times 100 \]  

(31)

($Q^s_t = Q^s$ when $Z = G_t - G_o$). We note, in particular, that

\[ Q^s \to 100 \text{ as } t \to \infty. \]  

(32)
Therefore, assuming $G_t \to \infty$ as $t \to \infty$, we deduce $Q_t^S \to 100$ as $t \to \infty$. The total reduction in $Q^S$ is $Q^S_t = 100$, and hence the percentage reduction at time $t$ for Scotland is:

$$D_t^S = \frac{Q_t^S - Q_o^S}{100 - Q_o^S} \times 100 = \frac{100}{10} \left( \frac{10}{10} - \frac{S_o}{G_o} \right) \left( \frac{G_t - G_o}{G_t} \right) \times 100 \times 100$$

$$= \frac{100}{10} \left( \frac{10}{10} - \frac{S_o}{G_o} \right) \frac{A_t}{1 - \frac{100}{10} \times \frac{S_o}{G_o}}$$

(34)

where $A_t = \left( \frac{G_t - G_o}{G_t} \right) \times 100$ is the cumulative increment as a percentage of the GB block. Therefore:

$$D^S = A_t.$$  

Equivalent arguments for Wales and England produce the identical result. Although this argument has been developed only with regard to Great Britain, it can be extended to Northern Ireland.

Case 2: 10:5:85 as rounded population shares

Here the calculations are much the same except now we do not assume that the ratio $P_S : P_W : P_E$ is the same as $b_S : b_W : b_E$ (=10:5:85). Indeed the second ratio was based initially on rounded population shares and so was not the same as the actual population ratio. From eq. (23) we obtain:

$$Q^S = \frac{P_a}{P_g} \left( \frac{G_o + b_g Z}{G_o + Z} \right) \times 100$$

(36)

(The $Q^S = Q_t^S$ when $G_t - G_o = Z$) and so:

$$Q^S \to \frac{P_a}{P_g} b_g \times 100 \text{ as } Z \to \infty.$$  

The calculation of $D^S$ is much the same as before:

$$D_t^S = \left( \frac{Q_t^S - Q_o^S}{P_g b_g \times 100 - Q_o^S} \right) \times 100$$

(37)

$$= A_t.$$  

The crucial distinction is that the limit for Scotland is different from the limits for Wales and England. Eq. (38) only applies to Scotland, though equivalent expressions can readily be derived for the other two territories, and the argument can then be again extended to Northern Ireland.
REFERENCES

Act of Union (Ireland) 1800, Chapter 38 (London: HMSO).
Darling, Alistair, (1997), Hansard, 9 December, Cols. 510-13W.
Education (Scotland) Act 1918, Chapter 48 (London: HMSO).
Goschen, George J., (1888a), Hansard, 26 March, cols. 300-301.
Goschen, George J., (1888b), Hansard, 23 April, col. 165.
Maclay, John, (1957a), Hansard, 12 February, col. 1087.
Mitchell, James, (1991), Scottish Public Finances (Glasgow: mimeo).
Teutemann, Manfred, (1993), "Interpersonal vs. Interregional Redistribution at the European Level – As Seen from the Perspectives of Fiscal Federalism and Public Choice Theory", *European Economy: Reports and Studies*, No. 5, pp. 395-413.
Walsh, Cliff, (1993), "Fiscal Federalism: an Overview of Issues and a Discussion of Their Relevance to the European Community", *European Economy: Reports and Studies*, No. 5, pp. 25-62.